DESIGN AND OPTIMIZATION OF A FAST HEAT PIPE THERMIONIC REACTOR 1

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THERMIONIC REACTOR I[*]

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ABSTRACT. The concept of an energy supply plant /61[***]
for space vehicles, consisting of a fast reactor as
heat source and out-of-core thermionic converters
heated and cooled by heat pipes, is designed and
optimized. The criterion for optimization is the
cost-to-power ratio. The costs are given by the
price of the fissionable material and the expense for
transport of the device into orbit. The reactor power
is limited by the maximum tolerable values for the
temperature at the free surface of a cell and for
the output current of a single converter. If the
composition of core and reflector and the converter
output are given, it is possible, applying the
reflector saving concept, to construct an analytical
correlation between the geometric parameters as well
as between these and the reactor power output. There-
fore it is possible to optimize these parameters with
a very small amount of calculations.

1. Introduction and Statement of the Problem

Development of space travel leads to increasingly higher requirements
on the power and lifetime of the power supply systems for spacecraft. For
powers greater than a few kilowatts, use of nuclear reactors in combination
with thermionic converters is planned [1, 2]. These systems are particularly
well suited for the mission established, because on one hand they exhibit a
relatively favorable conversion efficiency (ca. 10%) in spite of a high waste

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1

heat temperature (ca. 1,000° K), and on the other hand they have a high specific energy content (kWh/kg) and a high specific power (kW/kg).

The combination of the two system components, the reactor (heat source) and the converter (conversion of the heat into electrical energy) can be done in very different ways. Pruschek and co-workers have made detailed investigations of some possible combinations and compared the results [3]. At present, the arrangement with the converter within the fission zone of a thermal reactor, combined with liquid metal cooling, is considered to be the most favorable one which is technologically possible for production of electrical power in the range from about 10 to 100 kW under the restrictions of space travel. Development of the prototype of a system working on this principle is already in an advanced stage in Germany [4]. In spite of the resulting commitment to a certain line of development, it appears reasonable to outline other possible variants of a thermionic reactor also, and to discuss them, because the boundary conditions which at one time have led to a certain decision in this area are subjected to continuous change due to advancing technological developments. For instance, we can today consider the problem of heat transport at high temperatures (ca. 2,000° K) to have been solved through the development of suitable heat pipes. Thus it becomes possible to separate the heat source and the conversion portion in space, without having to give up intensive cooling of the fission zone. Rühle et al. [5] referred to this design principle as early as 1965.

This work covers the design and optimization of a fast thermionic low-power reactor with converters outside the fission zone, heated and cooled with heat pipes. The basic characteristics of such a system are:

1. The converter, because of its position outside the fission zone, can be designed according to pure converter physics viewpoints, as the neutron-physical properties of the materials are of no importance. Its operating safety is significantly greater, in comparison to in-core thermionic reactors, as neither poisoning of the electrodes by fission products nor change

2

of the electrode separation due to swelling of the nuclear fuel is possible. The electrical connection for the individual converters can be made as desired without any design problems, i.e., optimally. As electron emission is isothermal, the power density within a converter is independent of position.

2. By use of heat pipes for heat transport, we can avoid the use of pumps with their undesired side effects (weight, power consumption, perhaps lubrication and sealing problems, and gyroscopic effects). This type of heat transport requires only small temperature differences. In the undisturbed operating condition, the maximum fuel temperature is only a little above the emitter temperature. Within the individual converter, the power density and emitter temperature are independent of position, and can be freely selected within certain limits.

3. Only one converter can be driven per cell, if we proceed from the fact that at present no material is known which has adequate insulating properties at 2,000° K and is matched to the prevailing working conditions. Thus the use of this reactor type is limited to the lower power range (about 5 to 50 kW).

The last-mentioned point essentially differentiates the reactor concept submitted here from similar designs (e.g., those of Fiebelmann, Neu and Rinaldini [18] or of Loewe [7]) which presume the presence of a suitable insulator and which also, due to the type of construction, only come into consideration for powers above about 40 kW.

It is the goal of the optimization, to determine the energy supply installation having the described characteristics and which will supply a certain required electrical power at minimal costs.

The costs consist essentially of the following:

1. Costs for transporting the installation into space. It is assumed that these are approximately proportional to the installation weight.

3

2. Purchase price for the fission material U235. In contrast to thermal reactors, which have a relatively low fission material requirement, the costs of the fission materials represent a significant expenditure in the case of fast reactors.

Both cost factors[1] are certainly orders of magnitude higher than the raw material, manufacturing and assembly costs. Assuming that the latter are approximately proportional to the weight, they can formally be added to the transportation costs.

The parameters which influence the installation costs can be divided into two groups:

1. Parameters, the variation range of which extend over strictly defined limits because of physical, pyrometric and design reasons. By these we mean all material properties, geometric variables, such as for example wall thicknesses, current conduction cross sections and heat pipe dimensions. Therefore it does not make sense to vary these parameters. In general, we will limit ourselves to the selection of the most favorable solutions from the standpoint of the optimization criterion (Chapter 2: Design of the Installation).

2. Freely selectable geometric parameters. These are the distance across the cells, the dimensions of the fission zone, and the reflector thickness. These parameters are varied within technologically reasonable limits (Chapter 4: Optimization of the system described).

For determination of any particular optimal configuration of the parameters, the critical dimensions and the specific data for those dimensions (e.g., the charge of fissionable material, power, total costs, maximum fission zone

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[1] Estimated values see Chapter 5.1 in the following issue.

4

temperature) must be determined for a large number of reactors. Here, on one hand, the accuracy of the calculations and thus the reliability of the results should be as good as possible, and the computing expense on the other hand should be as small as possible. For parametric calculations, the relative accuracy is more important than absolute accuracy. The most accurate computation methods now available to determine the critical dimensions of reactors are based on the solution of the diffusion or transport equations in multidimensional geometries. But for even a single computation, they require so much computing time that parametric calculations cannot be carried out. Now the problem is one of developing a computing method which fulfills both of the requirements listed optimally (Chapter 3: Computation of the Critical Reactor Size).

As a criterion for accuracy, we use the results obtained with the DIFFU computer program (Solution of multigroup diffusion equations in two-dimensional goemetries) [25] for selected parameter values.

Figure 1 shows a simplified scheme for the course of the optimization.

We can check on the quality of the reactor structure and the materials used by means of criticality computations.

Freedom in design of the converter is limited by some boundary conditions prescribed by the structure of the fission zone. After establishing the converter operating conditions and calculating the Joule and thermal losses in the converter, we can determine the net electric power and the thermal /63 power of one cell. From this we obtain the amount of heat to be radiated away from the radiator. After selection of the radiation temperature, we obtain the required radiator surface.
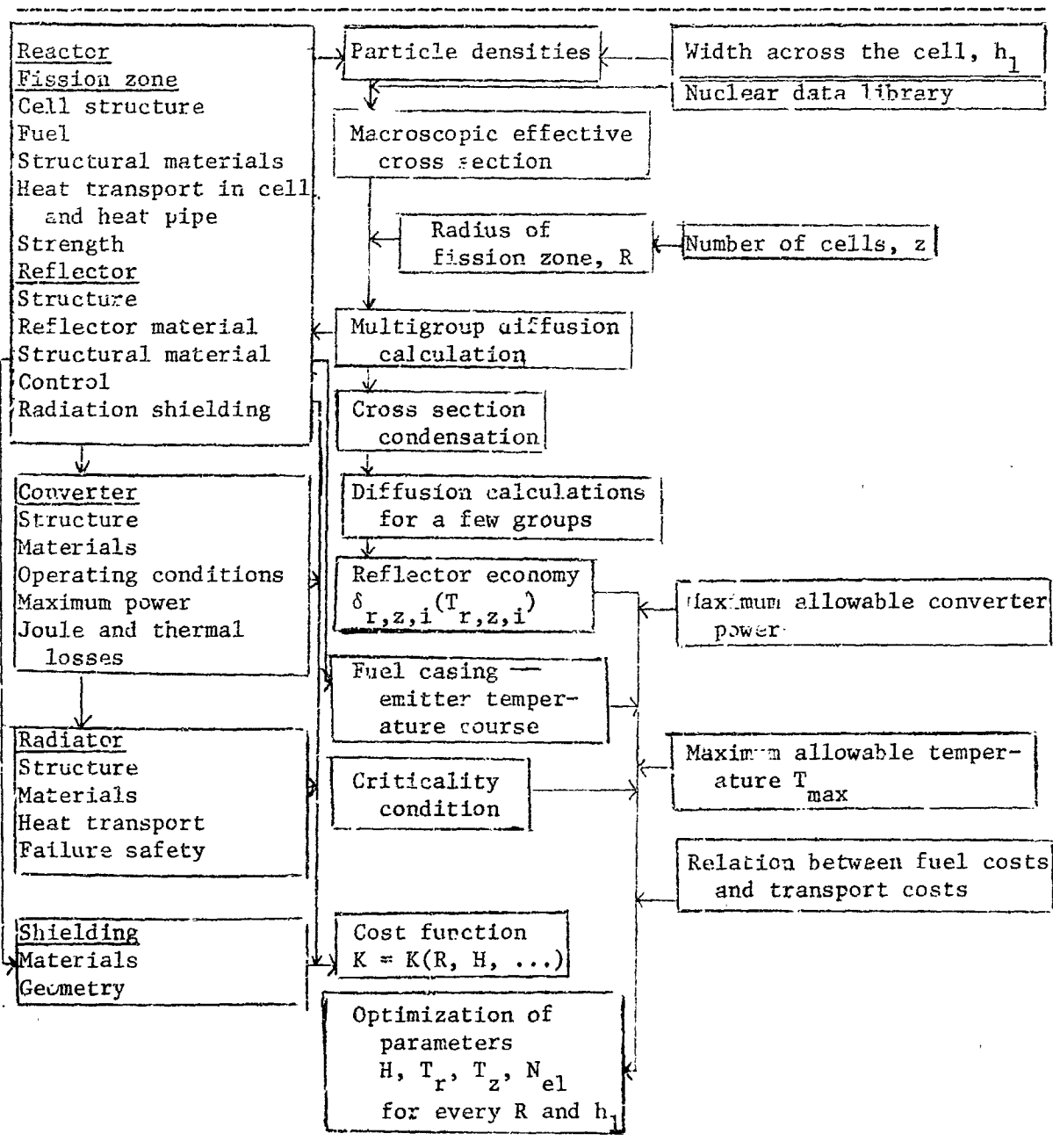
The dependence of the geometry for the shielding on that of the reactor is obvious.

Design and calculations   Computation   Prescribed data
for the design

| Reactor |
| Fission zone |
| Cell structure |
| Fuel |
| Structural materials |
| Heat transport in cell and heat pipe |
| Strength |
| Reflector |
| Structure |
| Reflector material |
| Structural material |
| Control |
| Radiation shielding |

Particle densities ← Width across the cell, h_1
Nuclear data library

Macroscopic effective cross section

Radius of fission zone, R ← Number of cells, z

Multigroup diffusion calculation

Cross section condensation

Diffusion calculations for a few groups

Reflector economy $\delta_{r,z,i}(T_{r,z,i})$ ← Maximum allowable converter power

| Converter |
| Structure |
| Materials |
| Operating conditions |
| Maximum power |
| Joule and thermal losses |

Fuel casing — emitter temperature course

Criticality condition ← Maximum allowable temperature T_{max}

| Radiator |
| Structure |
| Materials |
| Heat transport |
| Failure safety |

Relation between fuel costs and transport costs

| Shielding |
| Materials |
| Geometry |

Cost function $K = K(R, H, \ldots)$

Optimization of parameters H, T_r, T_z, N_{el} for every R and h_1

Graphic determination of the optimal parameter values and specific data for a prescribed N_{el}

Figure 1. Scheme of optimization.

6

If the practical design of the power supply system is established, then we can calculate the maximum temperature in the fission zone as a function of the free parameters and set up the cost function. We can also determine the functional relation between the reflector thickness and the reflector economy. For explanation of the neutron-physical calculations necessary for this, see Chapter 3. Because no analytical relation between the width across the cell and the cost function can be set up, optimization of the parameters breaks down into an analytical part and a graphical part. The maximum allowable values for the converter power and the fission zone temperature are boundary conditions for the optimization problem. The position of the optimum depends on the relation between fuel cost and transport cost.

2. Design of the Power Supply System

The statements in this chapter are essentially limited to presentation of the design principles, the criteria for the choice of materials, and the discussion of the technical data required for the optimization calculations (Chapters 3 and 4) which characterize the power supply system planned here. We have not presented the supporting calculations carried out in parallel with the design (e.g., strength and heat transport capability of the heat pipe, converter power and losses, working temperature of the mantle reflector, etc.). On one hand, the cost of doing so would exceed the limits of this report. On the other hand, variation of the design parameters in those respects is not reasonable on the grounds previously mentioned.

2.1. Reactor

Considering the geometry of the transport rocket, a power supply system for space vehicles should have the shape of a truncated cone. This requirement causes the individual components of the system to be arranged along an axis. As the payload must be protected by shielding from the neutron flux from the

reactor, but this shielding is one of the heaviest components, the opening angle of the cone must be kept as small as possible. Now, starting from the cylindrical reactor shape which is favorable for neutron economy, under the given conditions the axial arrangement of the converter and the division of the fission zone into cells parallel to the axis is prescribed.

2.1.1. Fission zone

If we first consider it apart from the problem of cooling the fission zone, the reactor power is independent of the size of the fission zone. Under these circumstances, application of the optimization criterion to a fast reactor is equivalent to the requirement for minimum critical size, i.e., maximum concentration of fissionable material and minimum proportion of structural material in the fission zone, as self-shielding has no effect here.

The most compact structure of the fission zone, with simultaneous optimal heat removal, is attained with a hexagonal shape for the cells and a central arrangement for the heat pipe. The converters must be electrically insulated from each other in order to allow a series-parallel connection[2]. At present there is no known material which has a sufficiently low rate of vaporization at temperatures around 2,000° K, as well as adequate insulating and strength properties. This produces the following decisive consequences for the structure of the reactor: 1. The cells can be insulated from each other only by an empty gap. 2. Only one converter can be driven per cell (see Figures 2 and 3).

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[2] As the electrical power from a converter appears at low voltage and current, but the Joule losses are proportional to the current, as many converters as possible should be connected in series, thus increasing the voltage of the current leading to the current-voltage converter, which must be located outside the shield. But on the other hand, insurance of the power generation against the result of failure of a single converter requires parallel connection. The series-parallel circuit is a compromise between the two opposing requirements. (in this respect, see [21]).
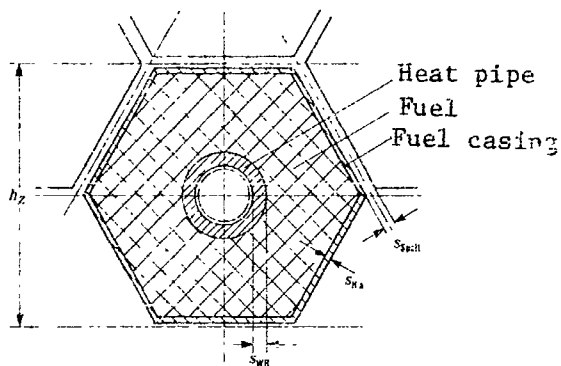
Heat pipe
Fuel
Fuel casing

h_z

S_{Spalt}

S_{K_A}

S_{WR}

Figure 2. Section across the reactor cell.

Fixed end
Reflector (shield side)
Adjacent cell
Fuel matrix
Heat pipe
End reflector
Collector with heat pipe structure
Converter
Electrical conductor with Cs line
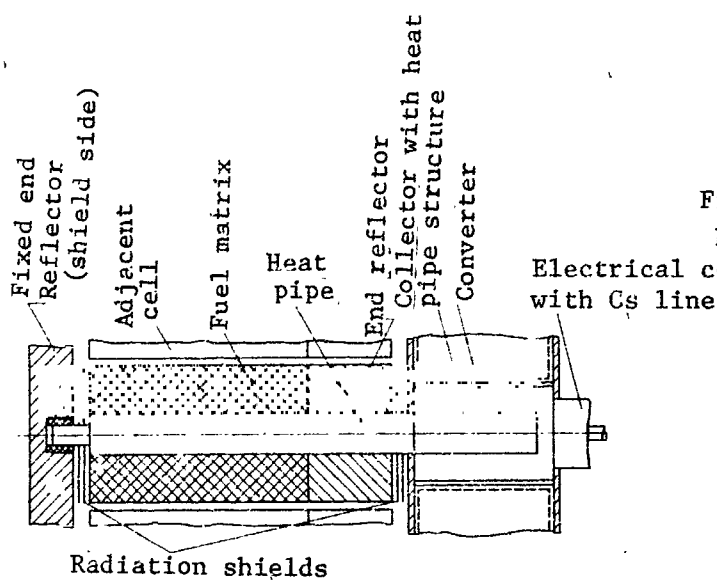
Radiation shields

Figure 3. Design showing the principle of the reactor cell.

The empty volume in the fission zone is the smallest with an arrangement having insulating gaps between the fuel and the heat pipe (as, for example, in [18]). But since the heat must be transported by radiation, this leads to very high fuel temperatures even with very small separation from the fission zone, because of the necessarily high heat flux density (some 50 to 150 W/cm^2). An arrangement with the gap between the emitter heat pipe and the emitter is not reasonable for the same reasons. Thus the first-mentioned limit on freedom of design produces a loosening of the reactor structure. The cell structure resulting from these considerations is presented in Figure 2.

The effect of the second limit is that this type of reactor is primarily suitable only for low powers, as an increase in the number of converters is possible only by increasing the radius of the fission zone, and limits are set to that by the acceptable diameter of the entire system.

The next step is the selection of the nuclear fuel and the structural materials. This step cannot be taken with enough certainty without extensive experimental investigations providing information on the behavior of the materials in the reactor under the given conditions. But the critical size of the reactor varies only slightly with a variation in materials. In the following, the selection is substantiated for the reactor concept under discussion here.

The following criteria apply to the fuel:

(a) thermal stability

(b) good thermal conductivity

(c) high atomic density for the fissionable isotope

(d) behavior under irradiation

Thus only UC or UN come under consideration as fuels, as UO_2, which is common and tested in thermal reactors, is excluded because of its low thermal conductivity. In the great majority of the previously published studies on fast thermionic reactors, UC is assumed to be the fuel. But UN has some important advantages: higher density, higher melting point, better thermal conductivity, lower coefficient of thermal expansion [1] , lower vaporization rate, and presumably better compatibility with the possible structural materials [8]. Unfortunately, there is still much uncertainty about the behavior of UN under irradiation.

Neither UN nor UC retain sufficient strength at temperatures around 2,000° K. For this reason they must be surrounded by a casing of high-melting

10

and compatible material. The following requirements are placed on this structural material:

(a) high-temperature strength and little creeping

(b) low vaporization rate

(c) compatibility with the fuel

(d) mechanical workability

(e) ductility, in order to be able to follow the swelling of the nuclear fuel

(f) availability

The neutron-physical characteristics are of secondary importance in fast reactors with a small proportion of structural material, as the capture cross-section of the metals under consideration is small in comparison to the fission cross section of U 235 in the fast energy range, and they do not significantly differ from each other.

Of the materials available, Mo and Nb cannot be used because of their high vaporization rates at a free surface [12] at the maximum temperatures which will appear (see Chapter 5.4). Rhenium is very expensive and difficult to work. There remain only Ta and W and their alloys. W shows very good strength properties, but is very hard to work and has only very slight ductility. Ta, in contrast, has too little strength at high temperatures. Thus we will use an alloy of Ta with 10% W (Ta10W). Its mechanical and physical-chemical characteristics at high temperatures are well known [9]. On one hand, it has nearly the strength of W; and on the other hand it is ductile and workable like pure Ta. The latter is particularly important in view of the hexagonal shape of the external casing of the fuel. In respect to compatibility with the nuclear fuels UN and UC, nothing is known. In case it is inadequate, there is a possibility of evaporating a layer of material compatible with UN on the fuel side of the casing (perhaps W or Re [8]).

The following criteria apply to the heat pipe — transport agent:

(a) ability to wet the wall, with slight solubility of the wall material in the transport agent

(b) sufficient but not excessive vapor pressure at the working temperature

(c) lower melting point at low vapor pressure

(d) high surface tension and high heat of vaporization.

The specified criteria are best fulfilled by lithium at the planned emitter temperature of 1,800° K. In laboratory tests, axial heat flows of 15 kW/cm^2 have been obtained with it [10]. With slight alloying, Ta or W are suitable as wall materials [10]. For the reasons mentioned above (high-temperature strength, ductility, workability) we shall therefore plan to use Ta10W here also. It may if necessary be coated with W. Thus the inner fuel casing and the heat pipe are a single component.

The materials selected for the fission zone and the reflector and the dimensions of the construction elements in the fission zone are summarized in Table 1. They yield the characteristic cell data given in Table 2 on variation of h_z.

TABLE 1. MATERIALS IN THE FISSION ZONE AND REFLECTOR; DIMENSIONS
OF THE CONSTRUCTION ELEMENTS IN THE FISSION ZONE

Fission zone

Fuel	U 235
	UN, 93% enrichment; 93% of theoretical density
Structural material	Ta10W/W
Heat pipe wall material	Ta10W/W
Heat pipe working medium	Li
Heat pipe diameter , inner/outer	8/11.2 mm

(Cont.)

12

TABLE 1. MATERIALS IN THE FISSION ZONE AND REFLECTOR; DIMENSIONS
OF THE CONSTRUCTION ELEMENTS IN THE FISSION ZONE

Heat pipe W coating, inner/outer	0.2/0.1 mm
Thickness of fuel casing	0.6 mm
inner W coating	0.1 mm
Space between two cells	1.5 mm
Maximum allowable surface temperature of the fuel casing	2,400° K
Emission coefficient of the fuel casing	0.6

Reflector:

	Reflector material	Structural material
Axial reflector		
converter side	BeO	Ta10W
shield side	BeO/Be	Nb
Reflector supplementation	C	Nb
Mantle reflector	Be	—

(Concluded)

TABLE 2. CHARACTERISTIC DATA FOR THE CELL ON VARIATION OF h_z.

Cell number	1	2	3	4	
Width across the cell, h_z (including gap)	2.5	3.0	3.5	4.0	cm
Volume proportions					
Fuel	6 .36	70.17	75.88	79.85	%
Structure	18.17	13.92	11.23	9.34	%
Empty space	20.47	15.88	12.89	10.81	%
U 235 density · 10^{-22}	1.726	1.973	2.134	2.246	1/cm³

2.1.2. Reflector

The function of the reflector is to scatter back into the fission zone the greatest possible portion of the neutrons diffusing out of the fission zone. A strong moderating effect is undesirable for a fast reactor, however, as the slowed and backscattered neutrons are effective only at the outer border of the fission zone, and produce an excessive fission rate there. We shall undertake selection of the reflector material according to the following criteria:

(a) large macroscopic scattering cross section

(b) low moderating capability

(c) low absorption

(d) low specific weight

(e) temperature resistance

The first four criteria are met within tolerable limits only by beryllium, Be, beryllium oxide, BeO, and graphite, C. From the viewpoint of neutron physics, Be is the most favorable and C the least favorable. One important reason for this is the n-2n reaction of Be in the fast energy range. This will increase the reactivity of our arrangement by about 1.5%, for example, with a reflector thickness of 10 cm.

The reflector consists of three parts:

1. End reflector, 1. Because the emitter heat pipe penetrates into it, its temperature is above 1,800° K. We assume that it has the same structure as the fission zone. The high vaporization rate of BeO requires the material to be jacketed. BeO is quite compatible with w [13], so we use the same casing as for the fuel.

2. End reflector, 2. This reflector is to consist of a fixed and a movable part. We shall mount the fuel rods in the fixed part. It is to be

protected against thermal radiation from the fission zone by radiation shields, and cooled by four radial heat pipes. As structural material, we use Nb 1 Zr, with Be as the reflector material. The movable part of the reflector, also made of Be, is used to control the reactor.

3. Mantle reflector. In order to compensate for the irregular external shape of the fission zone caused by the hexagonal shape of the cells, and to allow for a simple geometrical shape for the radiation shield, a supplementary reflector is required. We shall use pyrolytic graphite as the material for this, as it has considerably better workability than BeO. By means of the radiation shield it is possible to decrease the reflector temperature to the extent that use of Be becomes possible. In this way, we simultaneously decrease the heat loss from radiation of the surface into space.

2.2. Other System Components

Optimization of the other system components, the converter, radiator, and shielding (see Figure 5) is not, to be sure, an object of this work. But in order to be able to make valid statements about the possible electrical power and the weight of the system, we must have a conceptual design of all the components. We shall discuss these briefly in the following.

2.2.1. Converter

With the arrangement of the converter outside the fission zone, we have the ability to design it according to pure converter physics and heat technology requirements. We must consider only two limitations:

1. The diameter of the converter, including the collector cooling, is limited by the width across the cell, if we consider that the emitter heat pipe cannot be curved for manufacturing reasons.

2. To avoid an excessive voltage drop in the electrodes and in the leads to the current-voltage converter, which must lie outside the shielding, we

assume that the current at the converter output must not exceed a certain maximum value (see 4.1). With a given power density and converter voltage, then, the converter length is fixed.
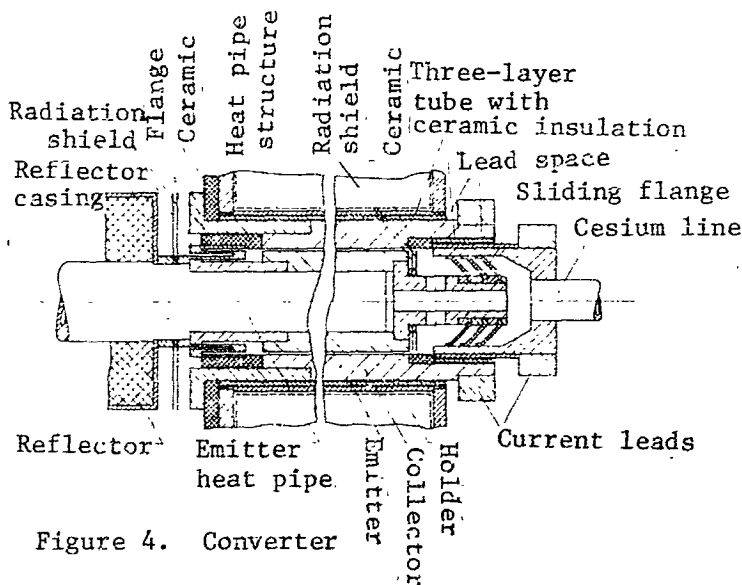

Figure 4. Converter

The converter design upon which we base the calculations is shown in Figure 4. As the emitter material, we plan to use Re vapor-coated with Mo. Re is distinguished by outstanding properties for converter physics, and Mo by good thermal and electrical conductivity. As its thermal expansion coefficient is smaller than that of the Ta 10W heat pipe material, good heat transfer is ensured by the high pressure between the two components at high temperatures. The collector material can be Nb 1 Zr. This has about the same thermal expansion characteristics as the Al_2O_3 used for insulation, so that thermal stresses are avoided. It has also proved good many times as a collector material as well as a heat pipe material in conjunction with sodium as the working medium (for collector cooling).

The electrical losses within the converter are decisively influenced by the dimensions of the converter electrodes. But decreasing the losses by /66 thickening the electrodes is counteracted by an increase in the temperature difference between the emitter heat pipe and the emitter surface or betwe the collector surface and the holder. The effect of the two effects on the optimization criterion must be balanced out.

The metal-ceramic junction between the emitter and the collector must in no case reach the melting temperature of the leads (about 1,200° C [14]). The

16

height and thickness of the connecting flange are to be designed accordingly, considering the strength required. For this reason we establish the collector temperature as 1,050° K, although a higher value would be more desirable because the radiator weight would then be less.

The junction between the emitter and the collector must be made movable so that we can compensate for the different thermal expansions of the two construction elements. The conductor cross section is to be optimized with the criterion of minimum power loss (thermal and Joule losses).

From the results of Wilson and Lawrence [16] on converters with wolfram emitters, and assuming that the maximum power density and the efficiency will increase by 30 to 50% on use of Re instead of W [17], we base the optimization calculations on the converter data listed in Table 3.

TABLE 3. CONVERTER DATA

Emitter/collector material	Re/Nb
Operating region	Volume ionization
Filling gas	Cesium
Emitter temperature	$T_E = 1,800°$ K
Collector temperature	$T_K = 1,050°$ K
Emitter current density	$j = 15$ A/cm^2
Average loss-free converter voltage	$U = 0.7$ V
Electrode separation	$s = 0.15$ mm
Converter operating efficiency (loss-free)	$\eta = 12\%$
Electrical power density at the emitter	$q_{el} = 10.5$ W/cm^2
Heat flux density at the emitter	$q_{th} = 87.5$ W/cm^2

The upper emitter temperature is limited by three boundary conditions:

(1) material compatibilities

(2) vapor pressure of the Li in the heat pipe

(3) maximum temperature in case of interruption (see 4.1.1)

Thus we establish it as 1,800° K, a relatively conservative value.

2.2.2. Radiator

Waste heat from the converter must be radiated into space by a radiator. Direct coupling of the converter with the radiator heat pipes according to [3] has basically the disadvantage that, if one radiator heat pipe fails, as from penetration of a meteorite, the related convertor is lost. Thus we shall transfer the heat from the collectors via an insulating layer (three-layer tube [15]) first to a heat exchanger with a heat pipe structure (arterial structure [6]). This then conducts the heat to the radiator heat pipes. In this design, even the loss of 10% of all the heat pipes has practically no effect on the converter operation. In this case, the collector temperature rises only by some 25 degrees at a radiator temperature of 1,000° K.

As the structural material for the converter and heat exchanger, we plan to use Nb 1 Zr, with Na as the working medium for the heat pipe.

2.2.3. Shielding

The shielding has the function of decreasing the radiation dose reaching the spacecraft payload to the extent that its radiation-sensitive components are not damaged during the time of the mission. Based on the results of shielding calculations which were performed for the ITR project [19], we shall consider a shield of 60 cm of lithium hydride to be adequate. As we have placed the radiator on the side of the reactor turned away from the payload (see Figure 5) and the direct thermal radiation from the reactor can
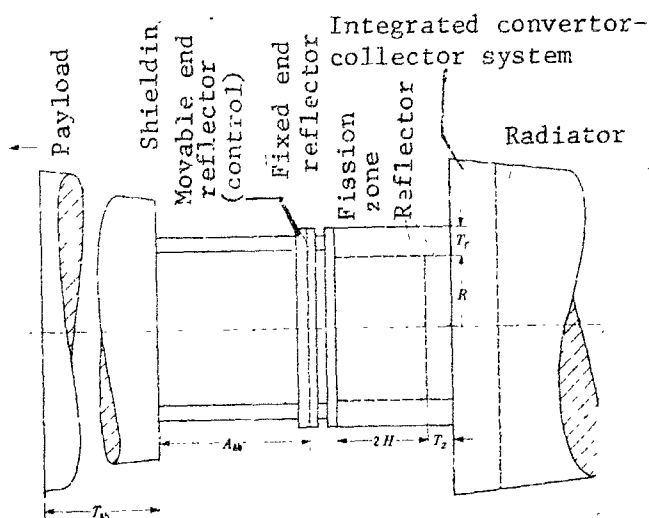
18

Figure 5. Structure of the power supply system.

be prevented by radiation shielding, the working temperature of the shield is far below the limit of thermal stress for LiH. Thus the use of high-temperature materials is not necessary. Back-scattering of radiation from the radiator in the direction of the payload is negligible because of the geometric $4\pi R^2$ weakening of the radiation and the distinct forward-scattering of the neutron and gamma radiation at the radiator [19]. Thus only the reactor itself must lie within the shadow cone of the shielding.

Because of the cone angle σ (here assumed to be 6°), the weight of the shielding increases with the distance from the reactor. Thus we make the distance as small as possible. As we shall use the end reflector turned toward the shield to control the reactor, the necessary apparatus for that (positioning and fast-cut-off system, control rods) must be arranged between the two structural components. For this we provide a distance of 30 cm. Change of this value within reasonable limits ($\pm$ 10 cm) has only a slight effect on the specific system cost.

3. Computation of the Critical Reactor Size

3.1. Diffusion Computation with the Computer Programs MUDIMA and DIFFU.

In the ordinary notation, the multigroup diffusion equations for $g = 1 \dots G$ are (see [24]):

19

$$\nabla D_g \nabla \Phi_g(x) \quad \Sigma_{T,g} \Phi_g(x) + \lambda \chi_g \sum_{g'=1}^{G} \nu \Sigma_{f,g'} \Phi_{g'}(x) \cdots$$
$$+ \sum_{g'=1}^{G} \Sigma_{S,g' \to g} \Phi_{g'}(x) + B_g^2 D_g \Phi_g(x) \quad 0 \tag{3.1}$$

The problem is treated unidimensionally. With the transverse buckling B_g^2, the flow curvature in the other coordinate directions is considered approximately in the form of pseudo-absorption. This presupposes the separability of the fluxes according to the position coordinates:

$$\Phi_g \cdots \Phi_{g,1}(x) \, \Phi_{g,2}(y) \tag{3.2}$$

In fast reactors, the separation conditions are well fulfilled, as Siegert [26] has already shown. The separated diffusion equation for $\Phi_{g,2}(y)$ is:

$$\Delta \Phi_{g,2}(y) + B_g^2 \Phi_{g,2}(y) = 0 \tag{3.3}$$

(with $y = r - x \cdot e_r$, r = position vector)

From this there follows for B_g^2, after integration over the volume and application of the Gaussian integral theorem

$$B_g^2 = -\frac{\int \nabla \Phi_{g,2}(y) \cdot dO}{\int \Phi_{g,2}(y) \cdot dV} \tag{3.4}$$

To calculate the critical reactor size of a rotationally symmetrical cylindrical reactor, Equation (3.1) must be solved in r-z geometry. Two different computer programs for this are available at the Institute for Nuclear Energetics. Both are based on the difference method [20]. One always seeks the smallest eigenvalue ($\lambda = 1/k_{eff}$) for which there exist non-trivial solutions of the system of equations.

1. MUDIMA [24] computes with only one position coordinate. We can consider the second coordinate direction through the pseudo-absorption B_g^2

[Equation (3.4)], where we must determine the flux curve by computation with this position coordinate to compute the integrals in (3.4). Thus we again need to know the flux curve in the first coordinate direction. This leads us to an iterative computation of B_g^2.

2. DIFFU [25] considers two position coordinates. Pre-selection of a transverse buckling and the requirement for separability of the neutron fluxes thus drop out also. But computation with many energy groups requires a great amount of computing time, as we cannot go below a certain number of supporting points for reasons of accuracy. Also, we must not exceed the memory capacity of our computer system. Thus there must be a limitation to a few energy groups, which is to some extent linked to a loss of accuracy (see 3.1.2).

3.1.1. Homogenization of the material region

Both computer programs require as input data the macroscopic effective cross sections of the individual material regions. These cross sections will be considered as constant within each region. As the average scattering path length is significantly greater than the cell dimensions for fast compact reactors, we are justified in considering the cells as homogeneous with respect to neutron physics, and thus in neglecting their heterogeneity.

In the computations which were performed, the fission region was considered as cylindrical, with the boundary inhomogeneities neglected. We likewise did not consider the different structures of the two end reflectors (see 2.1.2). We used the average particle densities of the converter side of the reflector for both construction components. The higher particle density for the shield side of the reflector ensures sufficient reserve reactivity for control of the reactor.

21

3.1.2. Determination of the cross section

The 26-group data set of Bondarenko [27] is usually used for computations for fast reactors. This data set contains the microscopic effective cross sections for all the materials to be considered in our calculations.

We perform the condensation of the cross sections to a few energy groups according to the following rule:

$$
\Sigma_{I,n} = \frac{\sum\limits_{g=g_u(I)}^{g_o(I)} \Sigma_{g,n} \int_{V_n} \phi_g \, dV}{\sum\limits_{g=g_u(I)}^{g_o(I)} \int_{V_n} \phi_g \, dV}
\tag{3.5}
$$

where $g_u(I)$ and $g_o(I)$ are, respectively, the lower and upper microgroups within the energy interval of macro-group I in the material region n.

The flux integrals are determined from computations with a position variable (MUDIMA program, see above). We undertake the division of the energy groups so that for a typical reactor arrangement the eigenvalues of a many-group computation and a few-group computation shall agree as exactly as possible with other parameters the same.

With the distribution selected, the eigenvalues differ by 1.5% to 2.5% (see Table 4). The four-group calculation yields a smaller eigenvalue and the difference from the reflector thickness becomes smaller.

TABLE 4. DISTRIBUTION OF THE NEUTRON SPECTRA INTO ENERGY GROUPS.

Macrogroup	Microgroups	Energy range
1	1 - 6	10.5 - 0.4 MeV
2	7 - 11	400 - 10 keV
3	12 - 19	10,000 - 21.5 eV
4	20 - 26	21.5 - 0 eV

3.2. Calculation According to the One-Group Theory

The simplest possible method for the computation of fast reactors, which is also generally used for parametric calculations (e.g., in [23]) is based on the analytical solution of the monoenergetic diffusion equation. If ϕ depends on only a single position coordinate, then there exist exact analytical solutions for the differential equations.

If the flux is separable according to the position coordinates, then, according to Meghreblian and Holmes [22], the criticality condition for a cylindrical reactor without end reflectors is:

$$B^2 = \left(\frac{\pi/2}{H + d_z}\right)^2 + \left(\frac{2,405}{R + d_r}\right)^2. \tag{3.6}$$

where H = half-height of the fission zone; R = fission zone radius; d = extrapolation length.

For the cylindrical reactor with reflectors at all sides there is no analytical criticality condition equivalent to (3.6). Therefore we must use the exact solutions of the equations which depend on only a single position coordinate, and consider the other position coordinates approximately through introduction of a pseudoabsorption.

The Algol program ANARZ [28] was made up for the equation system given by [22] for solution of the problem. The single-group effective cross section has been determined according to the method described in Section 3.1.2, for a mean reflector thickness of T = 6 cm. It appears that the differences from the DIFFU results, considered to be exact, are between -20% and -10%. The basic causes for this are:

1. The location dependence of the macrogroup cross sections, resulting from the slowing of the neutrons in the reflector and their backscattering into the fission zone, is completely neglected by averaging over the integral flux of the region and reduction to a single neutron group. Thus we

23

overestimate the fission source density in the center of the reactor.

2. Division of the mantle reflector into two regions with quite different diffusion characteristics is neglected by the homogenization. This becomes apparent especially with low reflector thicknesses (see Curve 3 in the lower family of curves, Figure 6).
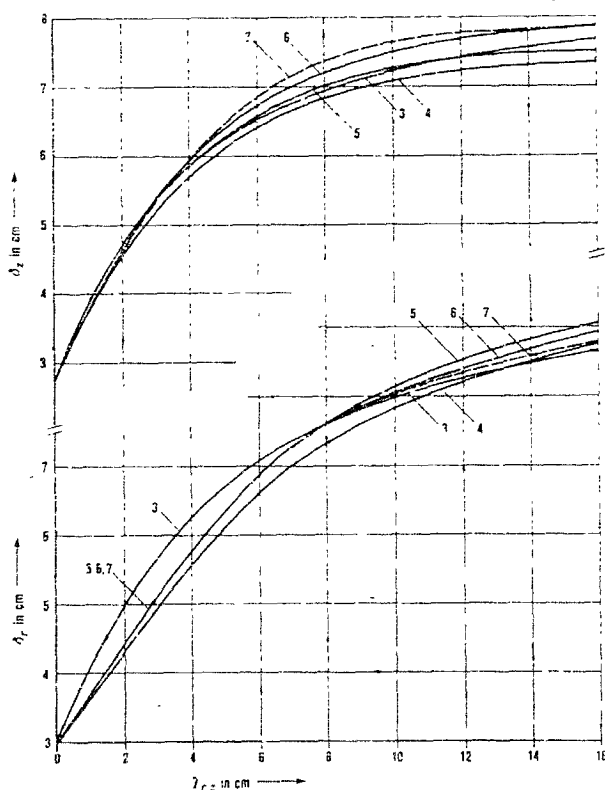

Figure 6. The function $\delta_i(T_{r,z\ i})$ obtained by different methods of calculation. The numbers are the same as those in Table 5. The curves are normalized to the initial value of the reference calculation ("DIFFU").

3. The choice of the magnitude of transverse buckling in the reflector is rather arbitrary, but it has a relatively large effect on the critical reactor size. This is shown by the discrepancy between the results in columns 3 and 4 of Table 5.

3.3 The Concept of Reflector Economy

Reflector economy is defined as the difference between the length of the bare fission zone including the extrapolation length, and the length of the fission zone of a reactor having a reflector.

$$\delta_r(T_r) = R_0 - R(T_r) = \frac{2,405}{B_0} - R(T_r) \qquad (3.7)$$
(cylinder)

$$\delta_z(T_z) = H_0 - H(T_z) = \frac{\pi/2}{B_0} - H(T_z) \qquad (3.8)$$
(plate)

24

where T_r, T_z are the radial and axial reflector thicknesses, respectively.

TABLE 5. DEVIATION OF THE CRITICAL FISSION REGION VOLUME FROM
COMPARISON VALUES WITH DIFFERENT CALCULATION METHODS
FOR $\delta_{r,z}$.

	1	2	3	4	5	6	7
$\frac{H_{DIFF} - H}{H_{DIFF}}$ (%)	- 18	10	- 2.6	9.2	- 2.8	0	- 0.3
	53	20	14	0.3	- 1.5	- 2.7	- 0.7

The index 0 refers to the bare fission zone. This definition includes no information about the flux curve or the distribution of the neutron spectra into energy groups used for the determination of R_0 and R or H_0 and H. By use of the same values for B_0 in Equations (3.7) and (3.8), we only assume that the spectrum of the bare reactor is independent of its geometrical shape.

With separability of the fluxes according to the position coordinates, the reflector economy is also independent of the flux curve in the other coordinate directions. Assuming that this independence also applies to the neutron spectrum, then we can assign a definite reflector economy $\delta_i(T_i)$ to a definite reflector thickness T_i for a given composition of the core and reflector. Now if we determine the δ_i for several T_i and approximate the curve of the function $\delta_i(T_i)$ by an analytical expression such as a polynomial, then the criticality condition

$$B_0^2 = \left(\frac{2.405}{R + \delta_r(T_r)}\right)^2 + \left(\frac{\pi/2}{H + \delta_z(T_z)}\right)^2 \tag{3.9}$$

represents an analytical relation between the parameters R, H, T_r and T_z.

If the critical height, H is calculated for various radii R at fixed T_r and T_z by means of DIFFU and we know B_0 from a critical calculation with a fission zone having no reflector, then we can determine the values for $\delta_r(T_r)$

25

and $\delta_z(T_z)$ by means of Equation (3.9). The results of other DIFFU computations can then be reproduced with (3.9) as an interpolation equation with a deviation of less than 1% (see Table 5, column 7).

Determination of the reflector economy by the single-group theory and the use of Equation (3.9) provides a clear improvement in the results as compared to the single-group calculation according to the transverse buckling concept (see 3.2 and Table 5, column 3). This is due to the fact that assumption of the transverse buckling in the reflector, which arbitrarily changes the relation between the absorption in the fission zone and that in the reactor, is superfluous.

There is an important source of error for computations with many energy groups in the fact that the neutron spectrum and, linked to it, the reflector economy, are not independent of the dimensions of the fission zone and the reflector. As long as the range of the back-scattered neutrons slowed by the reflector is small in the fission zone, in relation to the size of the latter, we can neglect this dependence. But this is no longer reasonable if the fission zone dimensions become very small. In this case, the calculated reflector economy is too small because of the softening of the spectrum, which results in decreasing the k_∞ of the fission zone. Agreement of the results from 26-group computations with the comparison values is good only at small reflector thicknesses, because here again the errors in determination of the cross section are the smallest. The displacement of the spectrum is considerably less if we calculate with a few energy groups for which cross sections were determined by quasi-two-dimensional calculations over the flux integrals. Then the neutron spectrum is practically stabilized already, and the reaction of the moderating properties of the reflector on the fission zone spectrum is considerably decreased (Table 5, column 5).

The limits of error can be narrowed even more if we consider the second coordinate direction through a transverse buckling. An exact computation for it is too expansive, however, while the assumption of a single lumped

26

value equally large for all groups and material regions leads to distortions of the absorption conditions and thus of the flux conditions between the individual groups. The choice of a relatively small transverse buckling, however, leads to a marked improvement (Table 4, Column 6). The values for the reflector economy calculated in this way and the fission zone dimensions resulting from it agree well with the comparison values (Figure 6, Curve 6). Therefore this method is used for determination of the reflector economy for the optimization computations

Table 5 gives the maximum and minimum deviations of the critical fission region from the comparison values from DIFFU computations for various calculation methods for $\delta_{r,z}$. The range of variation of reflector thickness is from 3 to 12 cm. The figures have the following meanings:

1. Single-group theory, $B^2_R = 0$

2. Single-group theory, $B^2_R = B^2_S = 0$
   ($B^2_{S,R}$ = transverse buckling in the fission zone or reflector, respectively)

3. δ concept, analytical single-group calculation, $B^2_R = B^2_S = 0$

4. δ concept, 26-group calculation, with 1 position coordinate, $B^2_R = B^2_S = 0$

5. δ concept, 4-group calculation, with 1 position coordinate, $B^2_R = B^2_S = 0$

6. δ concept, 4-group calculation, with 1 position coordinate, $B^2_R = B^2_S = 0.005$

7. δ concept, 4-group calculation, with two position coordinates, (DIFFU).

The averaging of the cross sections for methods 1 - 6 is done over the spectrum of an average reactor (T = 6 cm, H/R = 0.8).

(Continued in the next issue)

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28

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